Question 1. For A a number, consider the function f defined by
$$f(x) = \begin{cases} 2x + A & \text{if } x < 2 \\ 4x^2 - 8 & \text{if } x \geq 2. \end{cases}$$

For which A is f continuous?

Solution .:.

Since $2x + A$ and $4x^2 - 8$ are each continuous (for any A), we just need to make sure they are glued together correctly at $x = 2$. In particular, this means $2 \cdot 2 + A = 4 \cdot 2^2 - 8$, which is just to say $4 + A = 8$. This is equivalent to $A = 4$, and so 4 is the only value of A that makes f continuous.

Question 2. Consider the function $f(x) = x^2 + 5x$. Compute $f'(1)$ using a difference quotient.

Solution .:.

The difference quotient for f at 1 is the following:
$$\frac{f(1 + \Delta x) - f(1)}{\Delta x} = \frac{(1 + \Delta x)^2 + 5(1 + \Delta x) - (1^2 + 5)}{\Delta x} = \frac{(6 + 7\Delta x + (\Delta x)^2) - 6}{\Delta x} = 7 + \Delta x.$$

Hence the derivative of f at 1 is $\lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x} = \lim_{\Delta x \to 0}(7 + \Delta x) = 7$.